

1. The line l passes through the point $P(2,1,3)$ and is perpendicular to the plane Π whose vector equation is

$$\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = 3$$

Find

- (a) a vector equation of the line l , (2)

- (b) the position vector of the point where l meets Π . (4)

- (c) Hence find the perpendicular distance of P from Π . (2)



2.

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 1 \\ 0 & 5 & 0 \end{pmatrix}$$

- (a) Show that matrix \mathbf{M} is not orthogonal. (2)
- (b) Using algebra, show that 1 is an eigenvalue of \mathbf{M} and find the other two eigenvalues of \mathbf{M} . (5)
- (c) Find an eigenvector of \mathbf{M} which corresponds to the eigenvalue 1 (2)

The transformation $M : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix \mathbf{M} .

- (d) Find a cartesian equation of the image, under this transformation, of the line

$$x = \frac{y}{2} = \frac{z}{-1} \quad (4)$$



6. [In this question you may use the appropriate trigonometric identities on page 6 of the pink Mathematical Formulae and Statistical Tables.]

The points $P(3 \cos \alpha, 2 \sin \alpha)$ and $Q(3 \cos \beta, 2 \sin \beta)$, where $\alpha \neq \beta$, lie on the ellipse with equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

- (a) Show the equation of the chord PQ is

$$\frac{x}{3} \cos \frac{(\alpha + \beta)}{2} + \frac{y}{2} \sin \frac{(\alpha + \beta)}{2} = \cos \frac{(\alpha - \beta)}{2}$$

(4)

- (b) Write down the coordinates of the mid-point of PQ .

(1)

Given that the gradient, m , of the chord PQ is a constant,

- (c) show that the centre of the chord lies on a line

$$y = -kx$$

expressing k in terms of m .

(5)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



9.

$$I_n = \int(x^2 + 1)^{-n} dx, \quad n > 0$$

(a) Show that, for $n > 0$

$$I_{n+1} = \frac{x(x^2 + 1)^{-n}}{2n} + \frac{2n - 1}{2n} I_n \qquad (5)$$

(b) Find I_2 (3)



Question 9 continued

[Lined area for writing the answer to Question 9]

(Total 8 marks)

Q9

TOTAL FOR PAPER: 75 MARKS

END

